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A SEMI-RATIONAL NUCLEATE BOILING HEAT FLUX CORRELATION

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Abstract—Tien's model for nucleate boiling heat transfer is re-examined in the light of a recent experiment by Benjamin and Westwater. The study leads to a semi-rational modification of his prediction, which gives better correlation of available data.

NOMENCLATURE

- A, heater surface area;
- a, undefined, constant exponent of ΔT;
- B, constant of correlation in equation (11);
- b, undefined, constant exponent of n;
- C_d, drag coefficient defined by equation (9);
- D_b, diameter of a bubble as it departs from the heater surface;
- f, frequency of emission of bubbles from the heater surface;
- g, acceleration due to gravity;
- h, $\bar{h}$, local and average heater transfer coefficients, $q/\Delta T$ and $\bar{q}/\Delta T$, respectively;
- k, thermal conductivity;
- N_{Pr}, Prandtl number;
- N_{Re}, Reynolds number;
- n, active nucleation site density, sites/unit area;
- n_e, effective active nucleation site density for a single site;
- q, $\bar{q}$, local and average heat fluxes, respectively;
- r, radial co-ordinate measured horizontally from an active site;
- T, temperature;
- ΔT, wall superheat, $T_{\text{heater surface}} - T_{\text{saturated liquid}}$;
- u, radial liquid velocity.

Greek symbols

- α, intensity of stagnation flow, u/r;
- β, the dimensionless group, $\alpha/n\nu$;
- δ, δ_{th}, hydrodynamic and thermal boundary layer thickness, respectively;
- μ, liquid viscosity;
- ν, liquid kinematic viscosity, μ/ρ_f;
- ρ_f, ρ_g, liquid and vapor densities, respectively;
- σ, surface tension between a liquid and its vapor.

INTRODUCTION

A RECENT sequence of papers has unfolded a model for nucleate boiling heat transfer that appears to be correct in its essential features. In 1955, Yamagata *et al.* [1] measured the average heat flux, $\bar{q}$, the saturated temperature difference, ΔT, and the active site density, n, in boiling water, and correlated their results using the expression:

$$\bar{q} = \text{constant}\, \Delta T^a n^b \tag{1}$$

in which a and b were 3/2 and 1/4 respectively. In 1960, Kurihara and Myers [2] measured $\bar{q}$, ΔT, and n in a variety of liquids and correlated these data with

$$\bar{q} = 820\,(\mathrm{h})^{-1/3}\,[k(\rho_g/\mu)^{1/3}\,N_{Pr}{}^{-0\cdot 89}]\,n^{1/3}\Delta T. \tag{2}$$

Equation (2) was formed with the aid of dimensional analysis and accounts for fluid properties.

Zuber [3] recognized that (1) has basic significance since it gives plausible representation of the effect of surface roughness. Then he contended that heat transfer resulted from bubble-induced, boundary layer convection, and used the equation for laminar, flat-plate convection (with a redefined Reynolds' modulus) to get (1) with $a = 2$ and $b = 1/4$.

Tien [4] succeeded in writing an equation for boiling heat flux by forming a physical model and describing it analytically. He argued that each rising column of bubbles induces a rising flow of liquid in its wake. This flow is approximately an inverted stagnation flow that is laminar near the active nucleation site. Sibulkin's [5] equation:

$$\frac{\bar{h}r}{k} = 1{\cdot}32\,(N_{Pr})^{1/3}\left(\frac{ur}{\nu}\right)^{1/2} \tag{3}$$

describes heat transfer from the wall to such a flow. Since the inward radial velocity, u, diminishes as αr, where the constant of proportionality, α, describes the intensity of the flow, equation (3) becomes:

$$\bar{q} = 1{\cdot}32\,k(N_{Pr})^{1/3}\left(\frac{\alpha}{\nu}\right)^{1/2}\Delta T. \tag{3a}$$

Tien then assumed that α depended upon ν, the kinematic viscosity, and n. The dimensionless group, β, where:

$$\beta \equiv \frac{\alpha}{n\nu} \tag{4}$$

should therefore fully describe the flow field and be a universal constant.

Yamagata *et al.* also measured the average thermal boundary layer thickness, δ_{th}, in their experiments. Their results are used to evaluate β and thence to eliminate α/ν from (3a). For stagnation flow, δ_{th} is:

$$\delta_{th} = \frac{2{\cdot}44r}{N_{Pr}^{1/3}}\left(\frac{\alpha r^2}{\nu}\right)^{1/2}. \tag{5}$$

Elimination of α/ν between (5) and (4) yields an equation among β, δ_{th}, and n which gives $\beta = 2150$ upon comparison with Yamagata's data. Substitution of this value into (4), and the result into (3a) gives Tien's dimensionless prediction:

$$\bar{q} = 61{\cdot}3\,k(N_{Pr})^{1/3}n^{1/2}\Delta T. \tag{6}$$

The present study will offer certain rationalized empirical corrections for inaccuracies in the assumptions underlying equation (6). The corrections will lead to a vastly improved semi-rational equation among $\bar{q}$, n, and ΔT.

COMPARISON OF EQUATIONS (2) AND (6) WITH EXPERIMENTAL DATA

Kurihara and Myer's comparison of equation (2) with their own data is reproduced in Fig. 1 with three additions.

(a) The data of Yamagata *et al.* have been added since Kurihara and Myers did not originally consider them.

(b) An "effective site density", n_e, has been noted as a consequence of Tien's equation (4). Equation (4) implies that the intensity of flow (characterized by α) increases directly as n, since the gross flow induced by a site subtends less area as n increases. Presumably, these rising flows are fed by downcoming liquid sheets which form the boundaries between active site influence domains. The fact that $\bar{q} \sim n^{1/3}$ in (2) (instead of $\bar{q} \sim n^{1/2}$ as predicted by Tien) implies that the downcoming sheets actually feed the stagnation flows imperfectly, so that some of the

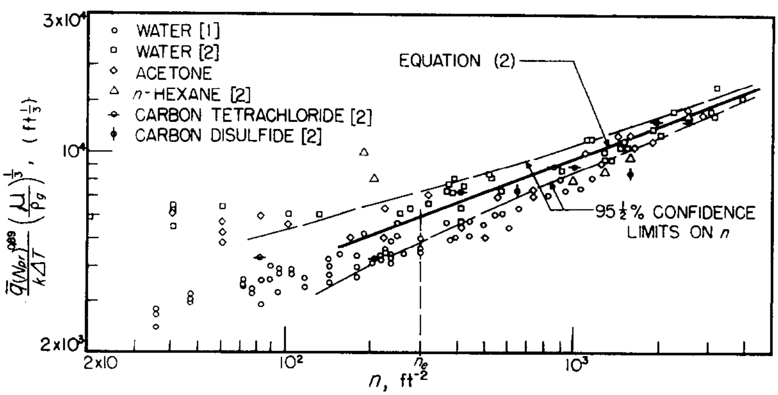

FIG. 1. Kurihara and Myers' correlation.

predicted reinforcement due to crowding of sites is lost.

When the influence area of an active site, n^{-1}, is greater than a limiting value, n_e^{-1}, the sites will cease to influence one another and (2) will cease to apply. In Fig. 2, $\bar{q}/k\Delta T(N_{Pr})^{-0.89}(\rho_g/\mu)^{1/3}$

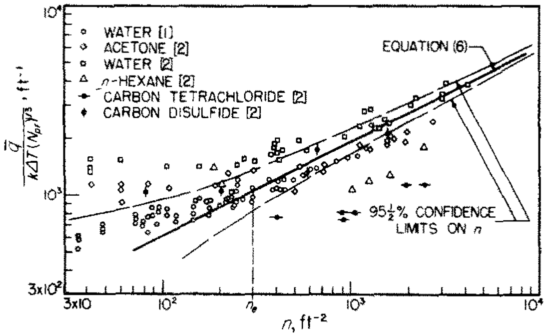

FIG. 2. Tien's correlation.

accordingly ceases to depend upon n when $n = n_e = 300 \pm 100$ sites/ft², or when the influence area is about 1/2 in². No variation of n_e with different liquids was apparent.

(c) An experimental measurement of n consists of sampling a supposed infinite heating surface for which $n = n_0$. $95\frac{1}{2}$ per cent of all measurements of n will fall within the two standard deviation limits of n_0 given by $n = n_0 \pm 2\sqrt{(n_0/A)}$ where A is the heater area, so long as $n_0 A \gtrsim 5$. These limits, based upon a 3 in diameter heater,* are included in Fig. 1. They reveal that (2) correlates roughly 80 per cent of Kurihara and Myers' own data within the $95\frac{1}{2}$ per cent confidence limits, but is somewhat less effective in correlating Yamagata's data.

Tien's comparison of (6) with both sets of data is reproduced in Fig. 2 with the $95\frac{1}{2}$ per cent confidence limits and apparent value of n_e again noted. n_e is still about 300 sites/ft², but the data for n-hexane, carbon tetrachloride and acetone fall outside of the $95\frac{1}{2}$ per cent limits. The success of (6)—which is very great indeed for a prediction of such a complicated phenomenon—is also impaired by the fact that $\bar{q}$ actually varies more nearly as $n^{1/3}$ than as $n^{1/2}$.

* Kurihara and Myers used a 3 in diameter plate; Yamagata *et al.* employed a slightly smaller (10 cm diameter) heater.

A DEFINITIVE EXPERIMENT

A recent paper by Benjamin and Westwater [6] provides a basis for correcting the inaccuracy of Tien's assumption that boiling heat transfer can be reduced to a *steady* convection process. While they sought to provide measurements of bubble growth in multicomponent systems, their results include q and ΔT data for pure water and glycol boiling at a *single active site*. Both q and ΔT were obtained locally at the site.

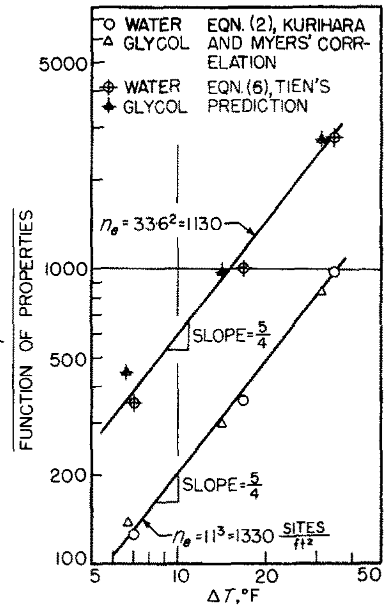

FIG. 3. Correlation of Benjamin and Westwater's heat transfer and temperature difference data.

These data have been correlated in Fig. 3 using both (2) and (3) in the following manner: each equation was regarded as a correlation between q and ΔT with the factor $n^{1/3}$ or $n^{1/2}$ assumed to be $n_e^{1/3}$ or $n_e^{1/2}$—the constant of correlation. The result was about the same in either case, namely:

$$\frac{q}{\text{function of properties}} = \left(\sim 1200\ \frac{\text{sites}}{\text{ft}^2}\right)^{1/3 \text{ or } 1/2} \Delta T^{5/4}. \quad (7)$$

Two conclusions follow as a consequence of equation (7):

(i) The two-dimensionality of heat flux beneath an active site is displayed by the fact that the local value of n_e is four times as great (or the linear dimension of the influence area, one half as great) as the average value. Accordingly q under the active site is greater than $\bar{q}$.

(ii) q varies not as ΔT, but as $\Delta T^{5/4}$. Therefore, $\bar{q}$ varies as some exponent between unity and 5/4, but weighted strongly in favor of the higher exponent. This is, in all likelihood, the result of local transient behavior in the liquid flow near the site, which Tien had to neglect, and which enhances heat transfer.

ON THE CONSTANCY OF β

The use of dimensionless analysis to form a supposed invariant dimensionless group β was based upon the assumption that average active sites in different boiling liquids were similar. They are not, however, since the product of the departure diameter of a bubble, D_b, and the frequency of emission, f, from an active site, is:

$$D_b f = \text{constant} \sqrt[4]{\left[\frac{\sigma g(\rho_f - \rho_g)}{\rho_f^2}\right]}. \tag{8}$$

(See, for example, Appendix B of [7])—a constant for one fluid, but a function of the fluid.

$D_b f$ has the physical significance of being the velocity at which bubbles would be rising if there were no distance between them. Since these bubbles induce flow through turbulent drag, one may heuristically propose that there exists a drag coefficient such that:

$$\begin{pmatrix}\text{The liquid head driving}\\ \text{stagnation flow}\end{pmatrix} = \left[C_d \rho_f \frac{(D_b f)^2}{2g}\right]\frac{1}{\rho_f} \tag{9}$$

in which C_d is constant if the flow is sufficiently turbulent. Then:

$$\text{the driving head} \sim \sqrt{\left[\frac{\sigma g(\rho_f - \rho_g)}{\rho_f^2}\right]}. \tag{9a}$$

Since the opposing drag of the laminar boundary layer varies as $(N_{Re})^{-1/2}$, the following correction suggests itself:

$$(N_{Re})^{1/2}{}_{\text{corrected}} = \left(\frac{\alpha r^2}{\nu}\right)^{1/2} \frac{\sqrt{[\sigma g(\rho_f - \rho g)/\rho_f^2]_{\text{actual}}}}{\sqrt{[\sigma g(\rho_f - \rho_g)/\rho_f^2]_{\text{reference}}}}. \tag{10}$$

Hence the boundary layer drag, as characterized by the inverse root of the Reynolds number, $\sqrt{(\nu/\alpha r^2)}$, is assumed to be reduced directly by (i.e. divided by) the increased driving effect of the fluid of interest over that of a reference fluid. Equation (10) must be offered as a primarily empirical compensation for the inconstancy of β, because the existence of a drag coefficient such as is described in (10) is conjectural.

FORMULATION AND TESTING OF CORRELATION

The present correlation can now be assembled by the simple expedient of altering Tien's prediction to accommodate the preceding observations that:

(1) $\bar{q}$ varies more nearly as $\Delta T^{5/4}$ than ΔT.
(2) $\bar{q}$ varies as $n^{1/3}$ instead of as $n^{1/2}$.
(3) The effect of the varying pumping capacity of bubble columns in different liquids must be accounted. One such accounting is proposed in equation (10).

The result is:

$$\bar{q}\,\frac{\text{Btu}}{\text{ft}^2\text{h}} = \left(\frac{\text{B}}{\text{ft}^{1/3}\ \text{degF}^{1/4}}\right)\left(k\,\frac{\text{Btu}}{\text{ft/h degF}}\right)(N_{Pr})^{1/3} \cdot \frac{\sqrt{[\sigma g(\rho_g - \rho_f)/\rho_f^2]_{\text{interest}}}}{\sqrt{[\sigma g(\rho_g - \rho_f)/\rho_f^2]_{\text{water}}}}(\Delta T\,\text{degF})^{5/4} \cdot \left(\frac{n}{\text{ft}^2}\right)^{1/3} \tag{11}$$

in which water has been selected as a reference fluid.

Fig. 4 displays the data of [1] and [2] subjected to correlation with equation (11). This correlation fixes the constant, B, as 110 $\text{ft}^{-1/3}$ $\text{degF}^{-1/4}$. Fluid properties were evaluated at the saturation temperature which was, in all cases, equal to the fluid bulk temperature.

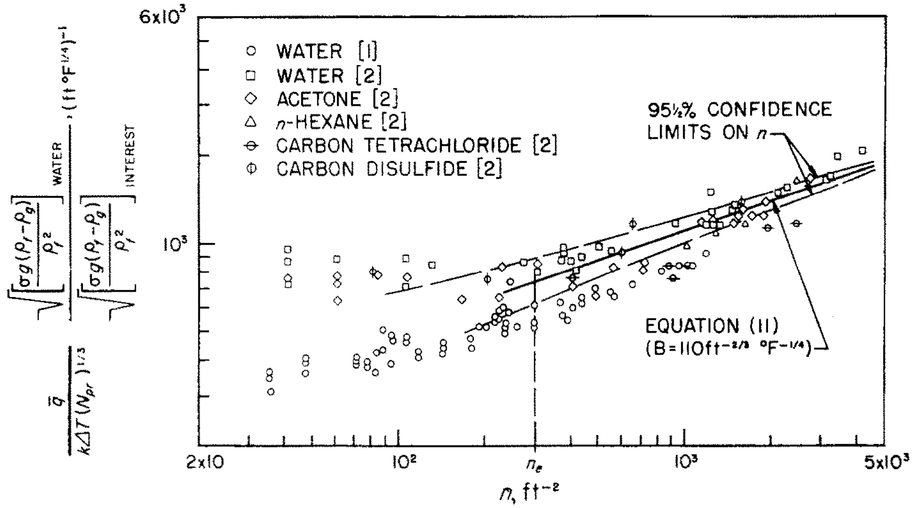

FIG. 4. The present correlation, equation (11).

Equation (11) provides great improvement over Tien's prediction while retaining the sense of rationale upon which it was based. It is, in fact, about as successful in bringing the data of Kurihara and Myers together as was their empirical correlation. The fact that the data of Yamagata and co-workers fall somewhat beneath the data of Kurihara and Myers in all three schemes of correlation suggests that there might be a systematic error in either of the two sets of data. Additional experimental measurements are needed, however, both to answer this question, and to test existing correlations over a greater range of physical properties.

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Résumé—Le modèle de Tien pour la transmission de chaleur par ébullition nucléée est reexaminé à la lumière d'une expérience récente de Benjamin et Westwater. L'étude conduit à une modification semi-rationnelle des résultats prévus par Tien qui améliore la corrélations avec les données existantes.

Zusammenfassung—Tien's Modell des Wärmeübergangs beim Blasensieden wurde auf Grund kürzlich durchgeführter Versuche von Benjamin und Westwater nachgeprüft. Die Untersuchung führt auf eine halbrationale Modifikation seiner Aussage, die eine bessere Korrelation der verfügbaren Daten ermöglicht.